Math 551 - Problem Set 1

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1. Because D is the midpoint of BC, we know that D, B, C are collinear. Therefore, we know that $D = (1-t)B + tC$ (by Theorem 1, page 25 of Pedoe). Likewise, we know that $E = (1-r)C + rA$. We also know that $\overline{DE} = E - D$, so $\overline{DE} = ((1-r)C + rA) - ((1-t)B + tC)$, but because both D and E bisect their respective segments, we know that $(1-t) = t = .5 = r = (1-r)$. So our equation for $\overline{DE} = E - D$ is simplified to $(.5C + .5A) - (.5B + .5C) = .5(A - B)$. So we have $E - D = .5(A - B)$, where $E - D$ is the vector from point D to point E, and $A - B$ is the vector from point B to point A. Our equation $E - D = .5(A - B)$ shows that these vectors are equal with the exception of a scalar constant, which means that they are parallel (which means the same is true for their corresponding line segments $\overline{DE}$ and $\overline{BA}$) . This scalar constant is .5, which means that their ratio is $1 : 2$, which also applies to their line segments. Therefore we have shown that $\overline{DE}$ and $\overline{BA}$ are parallel and their ratio is $1 : 2$. $\square$

2. Because we know that K is the midpoint of AB and L is the midpoint of DC, we know that we may write $K = (1-t)A + tB$ and $L = (1-r)C + rD$ (by Theorem 1 referenced above). But because AB is bisected by K and DC is bisected by L, we may reduce these (see previous problem for more thorough explanation) to $K = .5A + .5B$ and $L = .5C + .5D$. Thus, our vector $\overrightarrow{KL} = L - K = (.5C + .5D) - (.5A + .5B) = .5C + .5D - .5A - .5B = .5(D - A) + .5(C - B)$. Using $D - A = \overrightarrow{AD}$ and $C - B = \overrightarrow{BC}$ we have that $\overrightarrow{KL} = .5\overrightarrow{AD} + .5\overrightarrow{BC}$.

3. We choose two arbitrary points O_1 and O_2, where $P_1, ..., P_5$ and $\mu_1, ...\mu_5$ remain the same for both O_1 and O_2. So for O_1 we have $\mu_1\overrightarrow{O_1P_1} + ... + \mu_5\overrightarrow{O_1P_5} = \mu_1(P_1 - O_1) + ... + \mu_5(P_5 - O_1)$ but because $\mu_1 + ... + \mu_5 = 1$ we know that this is equal to $(\mu_1 P_1 + ... + \mu_5 P_5) - O_1$, but because this vector is with respect to O_1, we must add O_1 to obtain this vector starting at an origin which results in simply $(\mu_1 P_1 + ... + \mu_5 P_5)$, which is a vector with no dependence on O_1. If this operation is repeated for O_2, the identical vector $(\mu_1 P_1 + ... + \mu_5 P_5)$ is achieved, and so we conclude that there is no dependence on the choice of O. $\square$

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